The Latency Information Theory Revolution, Part I: Its Control Roots

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Abstract—The control roots of latency information theory (LIT) are reviewed in this first paper of a three papers series. LIT is the universal guidance theory for efficient system designs that has inherently surfaced from the confluence of five ideas. They are: 1) The source entropy and channel capacity performance bounds of Shannon's mathematical theory of communication; 2) The latency time (LT) certainty of Einstein's relativity theory; 3) The information space (IS) uncertainty of Heisenberg's quantum physics; 4) The black hole Hawking radiation and its Boltzmann thermodynamics entropy ***S*** in SI *J/K*; and 5) The author's 1978 conjecture of a structural-physical LT-certainty/IS-uncertainty duality for stochastic control. LIT is characterized by a four quadrants revolution with two mathematical-intelligence quadrants and two physical-life ones. Each quadrant of LIT is assumed to be physically independent of the others and guides its designs with an entropy if it is IS-uncertain and an ectropy if it is LT-certain. While LIT's physical-life quadrants I and III address the efficient use of life time by physical signal movers and of life space by physical signal retainers, respectively, its mathematical-intelligence quadrants II and IV address the efficient use of intelligence space by mathematical signal sources and of processing time by mathematical signal processors, respectively. The theoretical and practical relevance of LIT has already been demonstrated using real-world adaptive radar, physics and biochemistry applications. It is the objective of this paper to demonstrate that the structural dualities that are exhibited by the four quadrants of LIT are similar to those that were earlier identified by the author for the practical solution of stochastic control problems. More specifically, his 1978 conjecture of a structural-physical LT-certainty/IS-uncertainty duality between bit detection communication and deterministic quantized control problem solutions that led him to the discovery of a Matched Processors practical alternative to Bellman's Dynamic Programming.

Index Terms—Control, information space uncertainty, latency time certainty, communication channels, observation sensors, knowledge unaided adaptive radar

1. Introduction

This paper investigates the control roots of latency information theory [1]-[3] in three sections. In Section 2 essential motivation background on latency information theory (LIT) is advanced that reviews the three fundamental dualities found in LIT. They are: 1) The latency time (LT) certainty/information space (IS) uncertainty duality; 2) The IS-communication/LT-observation duality; and 3) The mathematical-intelligence/physical-life duality. Then in Section 3 the classical formulations of stochastic 'continuous' control problems are noted to yield for the special case of the linear quadratic Gaussian (LQG) problem [4] a mathematical separation between the controller gain design and the state estimator gain design. Unfortunately, however, these control formulations do not directly address real-world scenarios where the controlled processes are non-linear and their disturbances non-Gaussian. Thus in practical settings LQG control solutions do not match real-world processor scenarios, which in turn results in controller implementations whose performance often depart sharply from that predicted by the LQG control theory. Furthermore, when relays and/or digital controllers are used to generate control actions a further non-linear mathematical complication arises since the controls must be derived under the assumption of quantized levels. In Section 4, the last section of this paper, stochastic *quantized* control problem solutions are given that are predicated on a conjectured structural-physical LT-certainty/IS-uncertainty duality, and has the merit of being directly applicable to non-linear processes with non-Gaussian disturbances. This LT-certainty/IS-uncertainty duality conjecture will be found to be exactly the same as that of LIT, and led the author in 1978 to the discovery of a LT-certainty control methodology called Matched Processors [5]-[6]. More specifically, the LT-certainty Matched Processors architectures are similar to the IS-uncertainty Matched Filters architectures that are derived for optimum bit detections [7]. Moreover, Matched Processors has been found to offer a practical alternative to Bellman's Dynamic Programming which suffers of a 'curse of dimensionality' [8].

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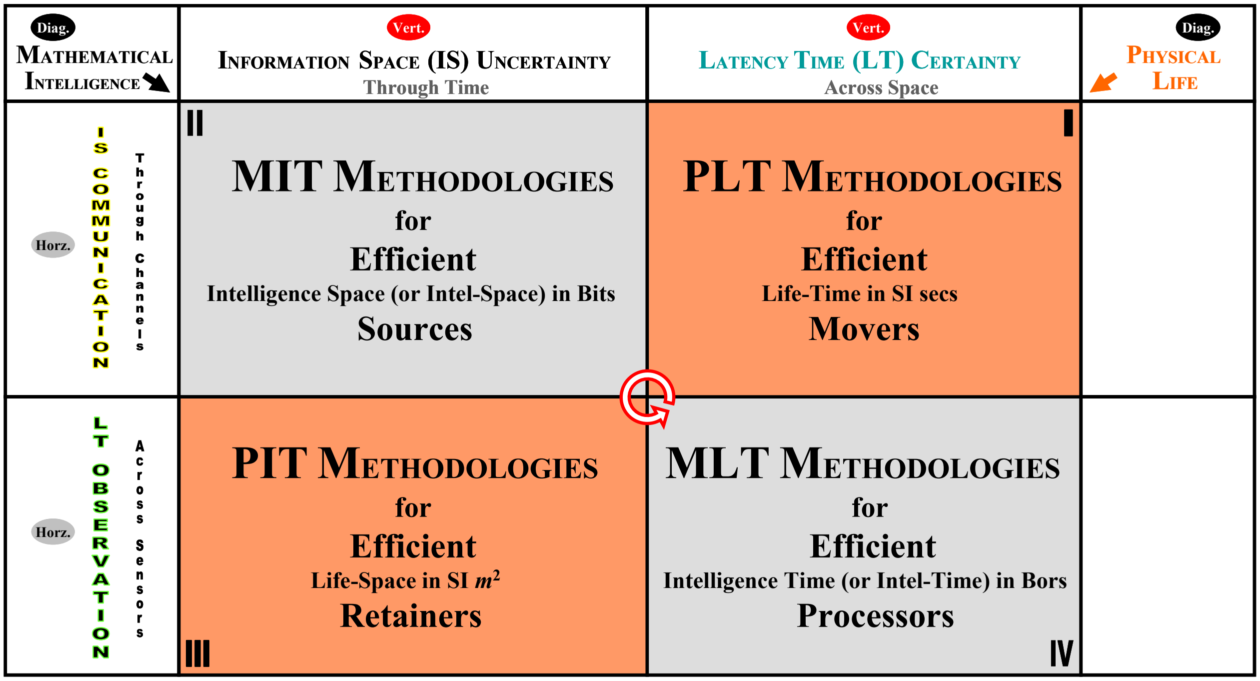


Fig.1. The Four Quadrants of the Latency Information Theory Revolution.

2. The Latency Information Theory Revolution

LIT is the universal guidance theory for efficient system designs that has inherently surfaced from the confluence of five ideas. They are: 1) The source entropy and channel capacity performance bounds of Shannon's mathematical theory of communication [9]; 2) The latency time (LT) certainty of Einstein's relativity theory; 3) The information space (IS) uncertainty of Heisenberg's quantum physics; 4) The black hole Hawking radiation and its Boltzmann thermodynamics entropy $\boldsymbol{S}$ in SI J/K [10]-[11]; and 5) The author's 1978 conjecture of a structural-physical LT-certainty/IS-uncertainty duality for stochastic control [5].

LIT is characterized by a four quadrants revolution with two mathematical-intelligence quadrants and two physical-life ones which are displayed in Fig. 1 and are reviewed next. In particular, classical information theory is in quadrant II and has been given the name mathematical information theory (or MIT) since the units of classical information are mathematical binary digits (bits). In addition, classical information is referred in LIT as sourced intelligence space (or intel-space in short). With this assigned terminology the distinct contribution of MIT to the structural-physical architectures and performance bounds of LIT will become transparent. More specifically, LIT has four system design guidance methodologies that are distributed in four quadrants which are assumed to be physically independent but are nevertheless bridged by statistical physics. The four quadrants are: 1) The IS-uncertainty/IS-communication MIT of quadrant II with its intel-space described with bit units and time-communicated through noisy intel-space channels; 2) The LT-certainty/LT-observation mathematical latency theory (MLT) of quadrant IV with its processing intelligence time (or intel-time in short) described with binary operator (or bor) units and space-observed across a window-limited intel-time sensor; 3) The IS-uncertainty/LT-observation physical information theory (PIT) of quadrant III with its retention life space (or life-space in short) described with SI square meter units (specifying the space surface area enclosing the retained signal) and time-observed across a noisy life-space sensor; and 4) The LT-certainty/IS-communication physical latency theory (PLT) of quadrant I with its motion life time (or life-time in short) described with SI second units (specifying the time delay of the moved signal) and space-communicated through a multi-path life-time channel.

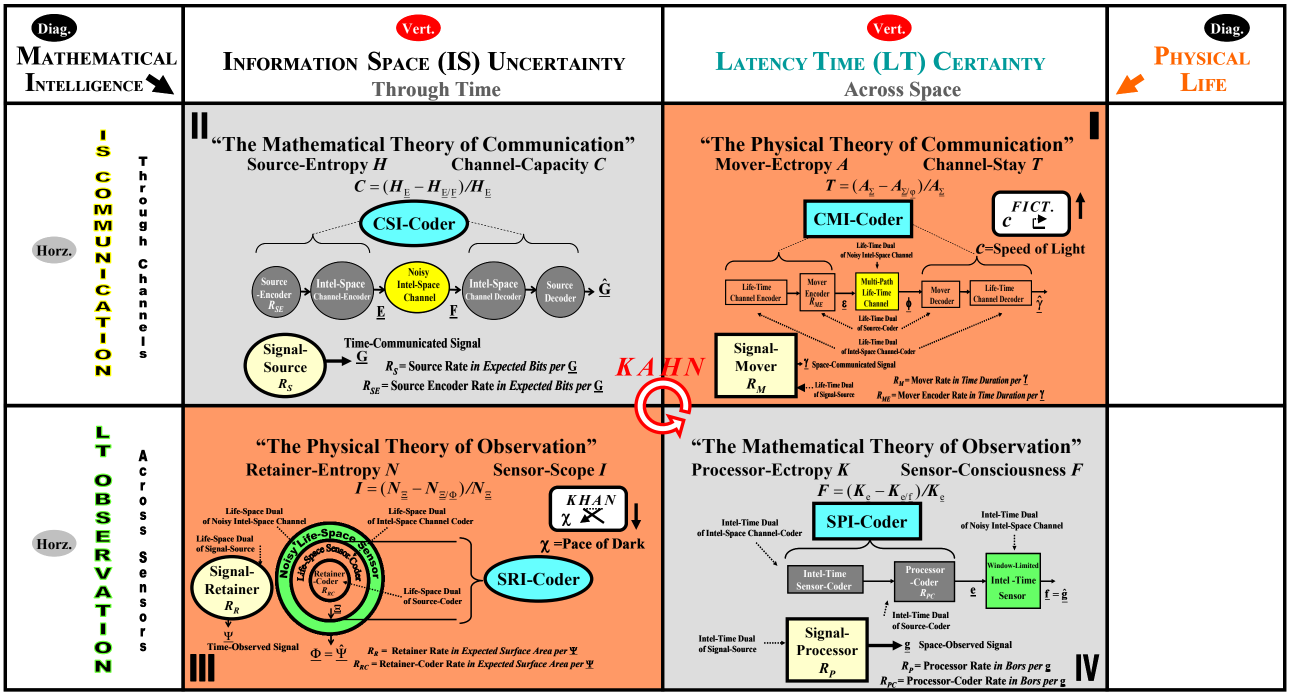


Fig. 2. The Structural-Physical LT-Certainty/IS-Uncertainty Dualities of the LIT Revolution.

Like MIT with its source-entropy and channel-capacity performance bounds, each of the three other system design methodologies, i.e., MLT, PIT and PLT, has two performance bounds that guide lossless and lossy system designs. Three major dualities are found in the LIT revolution. They are: 1) The LT-certainty/IS-uncertainty duality that is formed by the two certainty PLT and MLT schemes and the two uncertainty MIT and PIT schemes; 2) The physical-life/mathematical-intelligence duality that is formed by the two physical-life PLT and PIT schemes and the two mathematical-intelligence MIT and MLT schemes; and 3) The IS-communication/LT-observation duality that is formed by the two IS-communication PLT and MIT schemes and the two LT-observation PIT and MLT schemes. Also six minor dualities are noted form Fig. 1. They are the two minor PLT/MIT and MLT/PIT dualities of the LT-certainty/IS-uncertainty major duality then the two minor PLT/PIT and MIT/MLT dualities of the physical-life/mathematical-intelligence major duality and finally the two minor PLT/MLT and MIT/PIT dualities of the IS-communication/LT-observation major duality.

In Fig. 2 a display is given of the structural-physical LT-certainty/IS-uncertainty LIT dualities [1]-[3]. Some of the highlights of these structures are briefly discussed next to motivate their connection to their control roots which is the main topic of this paper:

1) The channel and source integrated (CSI) coder structures of the MIT of quadrant II are noted to have dual structures in each of the other three quadrants. More specifically, the PLT of quadrant I exhibits channel and mover integrated (CMI) coder structures with components that are the duals of the components of the CSI coder, e.g. the CMI coder is noted to have a mover encoder/decoder that is the life-time dual of the source encoder/decoder of the CSI coder. The same can be said for the sensor and retainer integrated (SRI) coder structure of the PIT of quadrant III and the sensor and processor integrated (SPI) coder structure of the MLT of quadrant IV.
2) The signal source that is replaced with the efficiency CSI coder is noted to have duals in each of the remaining three quadrants. They are: 1) the signal mover of the PLT of quadrant I that is replaced with the efficiency CMI coder; 2) the signal retainer of the PIT of quadrant III that is replaced with the efficiency SRI coder; and 3) the signal processor of the MLT of quadrant IV that is replaced with the efficiency SPI coder.
3) The source-entropy ***H*** (or expected source-information) of MIT that is used as a lower performance bound for lossless source-coder design is seen to have a dual in each of the remaining three quadrants. They are: 1) The mover-ectropy ***A*** (or minimax mover-latency) used as a lower performance bound for lossless mover-

coder design; 2) The retainer-entropy ***N*** (or expected retainer-information) used as a lower performance bound for lossless retainer-coder design; and 3) The processor-ectropy ***K*** (or minimax processor-latency) used as a lower performance bound for lossless processor-coder design.

4) The dimensionless channel-capacity ***C*** used as an upper performance bound for lossless CSI-coder design is seen to have a dual in each of the remaining three quadrants. More specifically, PLT exhibits a dimensionless channel-stay ***T*** as an upper performance bound for lossless CMI-coder design, PIT exhibits a dimensionless sensor-scope ***I*** as an upper performance bound for lossless SRI-coder design, and MLT exhibits a dimensionless sensor-consciousness ***F*** as an upper performance bound for lossless SPI-coder design. As a memory aid for the symbols used for these four performance bounds of LIT, it is noted from quadrant I of Fig. 2 that the contraction for the word fictional, i.e., ***FICT.***, surfaces while forming the shape of the speed of light symbol ***c*** (corresponding to the maximum possible speed for any signal mover of quadrant I), when one moves from quadrant to quadrant on the LIT revolution following the *upper* performance bound sequence ***F*** (quadrant IV)→***I*** (quadrant III)→***C*** (quadrant II)→***T*** (quadrant I).

5) While the movers of the physical quadrant I cannot exceed the speed of light in a vacuum limit of c=2.9979 x 10^8 *m/sec* conjectured by Einstein, the retainers of the physical quadrant III cannot exceed the pace of dark in a uncharged non-rotating black hole (UNBH) limit conjectured by the author and derived in [12] of χ=6.1123 x 10^{63} *sec*/*m*^{3}. As a memory aid for the symbols used for the four lower performance bounds of LIT, it is noted from quadrant III of Fig. 2, that the name ***KHAN*** surfaces while forming a similar shape as the pace of dark symbol χ by moving from quadrant to quadrant following the *lower* performance bound sequence ***K*** (quadrant IV)→***H*** (quadrant II)→***A*** (quadrant I)→***N*** (quadrant III)—it is also assumed here, of course, that the movement from ***H*** to ***A*** is not visible since it occurs at the speed of light ***c*** of quadrant I. Finally, the contraction ***FICT*** (***c***) of quadrant I for the LIT upper performance bounds, and the name ***KHAN*** (χ) of quadrant III for the LIT lower performance bounds, can be easily recalled together by thinking of 'a fictional ***FICT*** emperor ***KHAN*** that while wearing his crown ***c*** sits on his imperial chair χ'.

Moreover, the performance bounds of LIT can be bridged by thermodynamics and its recently discovered time dual that has been given the name lingerdynamics [1]-[2].

3. The Stochastic Linear Quadratic Gaussian (LQG) Control Problem

In Fig. 3 the discrete-time stochastic LQG control problem is displayed where: 1) The control system consists of a controlled signal processor (or plant), a noisy channel, and a controller with a sensor of the estimated state whose latency time window of observation is limited by the linear quadratic *continuous* control evaluator subsection; 2) The signal processor is modeled with a real discrete time linear state equation $\mathbf{x}_{k+1}$=$\mathbf{A}_k\mathbf{x}_k$+$\mathbf{B}_k\mathbf{u}_k$+$\mathbf{w}_k$ where $\mathbf{x}_k$ is a n-dimensional state vector, $\mathbf{u}_k$ is a p-dimensional input vector, the state matrix $\mathbf{A}_k$ and control vector $\mathbf{B}_k$ are of appropriate dimensions, and $\mathbf{w}_k$ is a n-dimensional vector denoting a zero mean Gaussian state noise; 3) The output system is modeled with the real linear equation $\mathbf{y}_k$=$\mathbf{C}_k\mathbf{x}_k$+$\mathbf{v}_k$ where $\mathbf{y}_k$ is a m-dimensional output vector, $\mathbf{C}_k$ is an output matrix of appropriate dimensions, and $\mathbf{v}_k$ is a m-dimensional disturbance vector denoting a zero mean Gaussian output noise; 4) The performance criterion (or cost to go) $J_{k,N} = E\left(\Sigma_{i=k,..,N-1}(\mathbf{x}_i^t Q_i \mathbf{x}_i + \mathbf{u}_i^t R_i \mathbf{u}_i) + \mathbf{x}_N^t F \mathbf{x}_N\right)$, $F \geq 0,\ Q_i \geq 0, R_i > 0$ is the expected value of the

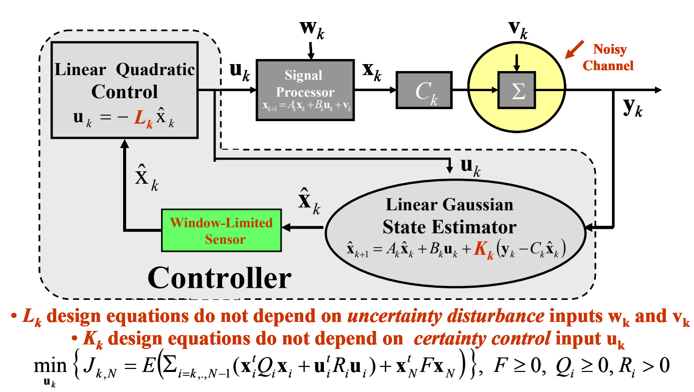


Fig. 3. The Discrete-Time Stochastic Linear Quadratic Gaussian (LQG) 'Continuous' Control Problem.

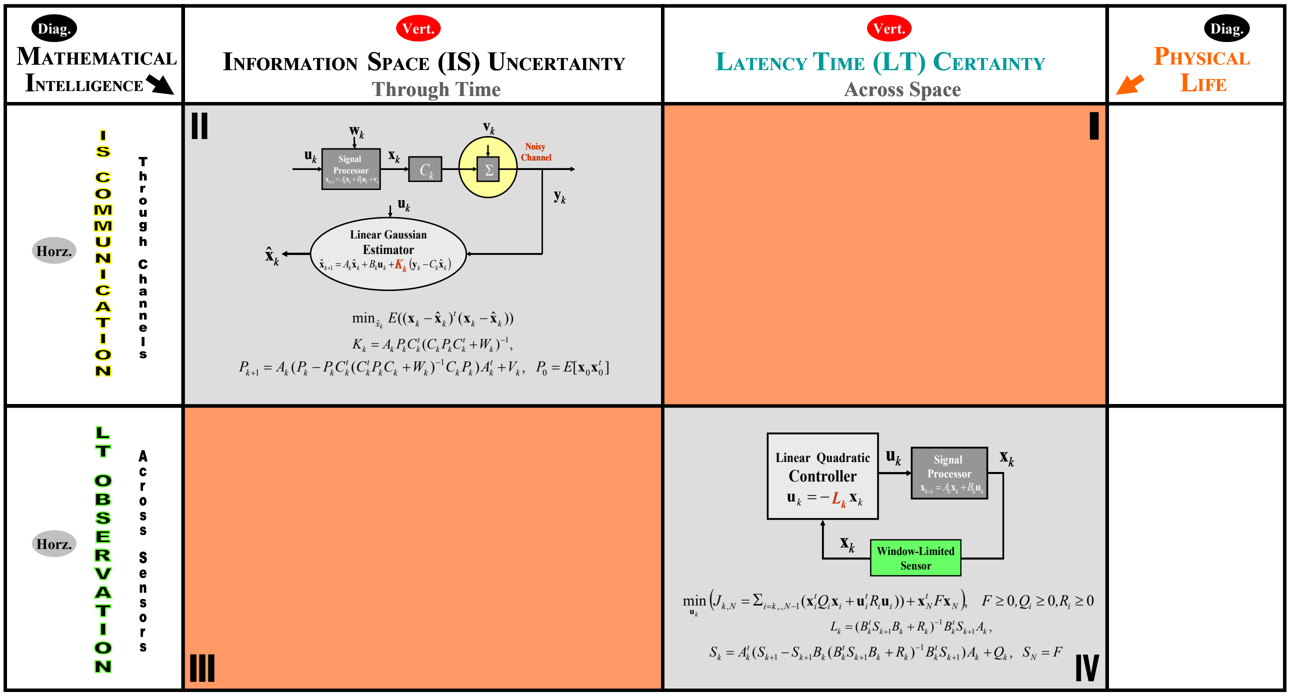


Fig. 4. The LIT revolution display of the IS-uncertainty LG estimation and LT-certainty continuous LQ control duals.

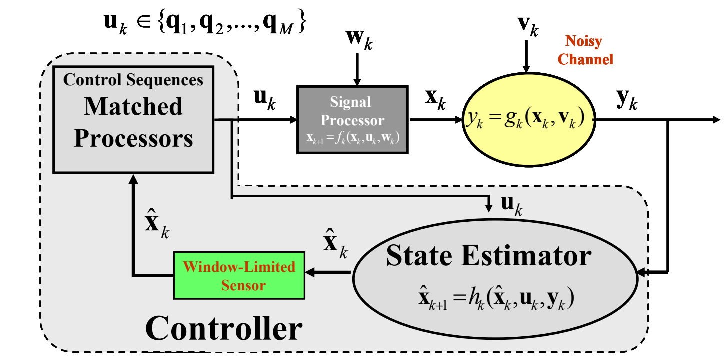
sum of the costs associated with the present and future states and controls where the matrices F, $\{Q_i\}$ and $\{R_i\}$ are weighting matrices of appropriate dimensions and properties linked to the state and control actions; and 5) the controller consists of the cascade of a state estimator and a continuous (or non-quantized) control evaluator whose control action $\mathbf{u}_k$ minimizes the cost to go.

Moreover, in Fig. 3 the solution to the LQG control problem is also noted where: 1) The state estimator is given by the real linear state equation $\hat{\mathbf{x}}_{k+1} = A_k\hat{\mathbf{x}}_k + B_k\mathbf{u}_k + K_k\left(\mathbf{y}_k - C_k\hat{\mathbf{x}}_k\right)$ with $\hat{\mathbf{x}}_k$ being an estimate of $\mathbf{x}_k$ and K_k is a gain matrix of appropriate dimensions that is designed off-line and is independent of the 'certainty' control input $\mathbf{u}_k$. In particular, K_k is found from the solution of a Riccati equation solved forwards in time [4]; 2) The control subsection is given by the real linear expression $\mathbf{u}_k$=$L_k\hat{\mathbf{x}}_k$ with L_k being a gain matrix of appropriate dimensions that is designed off-line and is independent of the 'uncertainty' disturbance inputs $\mathbf{w}_k$ and $\mathbf{v}_k$. In particular, L_k is found from the solution of a Riccati equation solved backwards in time [4].

The aforementioned separation of the L_k and K_k designs can also be viewed as the solution of *two* physically independent problems, one an LT-certainty control problem and the other an IS-uncertainty state estimation one. These two problems form together the structural-physical LT-certaint/IS-uncertainty duality depicted in Fig. 4 with the aid of the four quadrants LIT revolution. In Fig. 4 the time-dual Riccati equations used to find L_k and K_k are also stated along with the certainty and uncertainty quadratic performance criterions that when minimized give rise to them.

4. The Stochastic Matched Processors Control Problem

In Fig. 5 the discrete-time stochastic quantized control problem is displayed where: 1) The control system consists of a controlled signal processor, a noisy channel and a controller with a window-limited sensor of the state estimator output that satisfies the latency time observation constraint of the *quantized* control evaluator subsection; 2) The controlled signal processor is modeled with the real discrete nonlinear state equation $\mathbf{x}_{k+1}$=$\boldsymbol{f}_k(\mathbf{x}_k,\mathbf{u}_k,\mathbf{w}_k)$ where $\boldsymbol{f}_k$ is a



$$\min_{\mathbf{u}_k}\left\{J_{k,N} = E\left(h_{k,N}(\mathbf{X}_{k,N} = [\mathbf{x}_k,..,\mathbf{x}_N], \mathbf{U}_{k,N-1} = [\mathbf{u}_0,..,\mathbf{u}_{N-1}])\right)\right\}$$

Fig. 5. The Discrete-Time Stochastic 'Quantized' Control Problem.

nonlinear function of appropriate dimensions, $\mathbf{x}_k$ is a n-dimensional state vector, $\mathbf{u}_k$ is a quantized p-dimensional input vector with M possible realizations selected from the vector set $\{\mathbf{q}_1,..,\mathbf{q}_M\}$ and $\mathbf{w}_k$ is an arbitrary n-dimensional state noise; 3) The output system is modeled with the real nonlinear equation $\mathbf{y}_k = \boldsymbol{g}_k(\mathbf{x}_k, \mathbf{v}_k)$ where $\boldsymbol{g}_k$ is a nonlinear function of appropriate dimensions, $\mathbf{y}_k$ is a m-dimensional vector output and $\mathbf{v}_k$ is a m-dimensional vector denoting an arbitrary n-dimensional output noise; 4) The performance criterion (or cost to go) $J_{k,N}\left(\mathbf{x}_k, \mathbf{u}_k, \mathbf{U}_{k+1,N-1} = [\mathbf{u}(k+1),.,\mathbf{u}(N-1)]\right) = \sum_{i=k,.,N-1} L_i(x(i), u(i)) + L_N(x(N))\Big|_{\{\mathbf{x}_{k+1} = f_k(\mathbf{x}_k, \mathbf{u}_k)\}}$, is the sum of the costs $\{L_i\}$ associated with the present and future states and controls where $\mathbf{U}_{k+1,N-1} = [\mathbf{u}(k+1),.,\mathbf{u}(N-1)]$ denotes the future control sequence; 4) the controller consists of the cascade of a nonlinear state estimator $\hat{\mathbf{x}}_{k+1} = h_k(\hat{\mathbf{x}}_k, \mathbf{u}_k, \mathbf{y}_k)$ with $\hat{\mathbf{x}}_k$ being an estimate of $\mathbf{x}_k$ and a quantized control evaluator whose control action $\mathbf{u}_k$ is extracted from the countable set $\{\mathbf{q}_1,..,\mathbf{q}_M\}$ that minimizes the cost to go; and 5) the control evaluator has a structure made of processors that are matched to future control sequences.

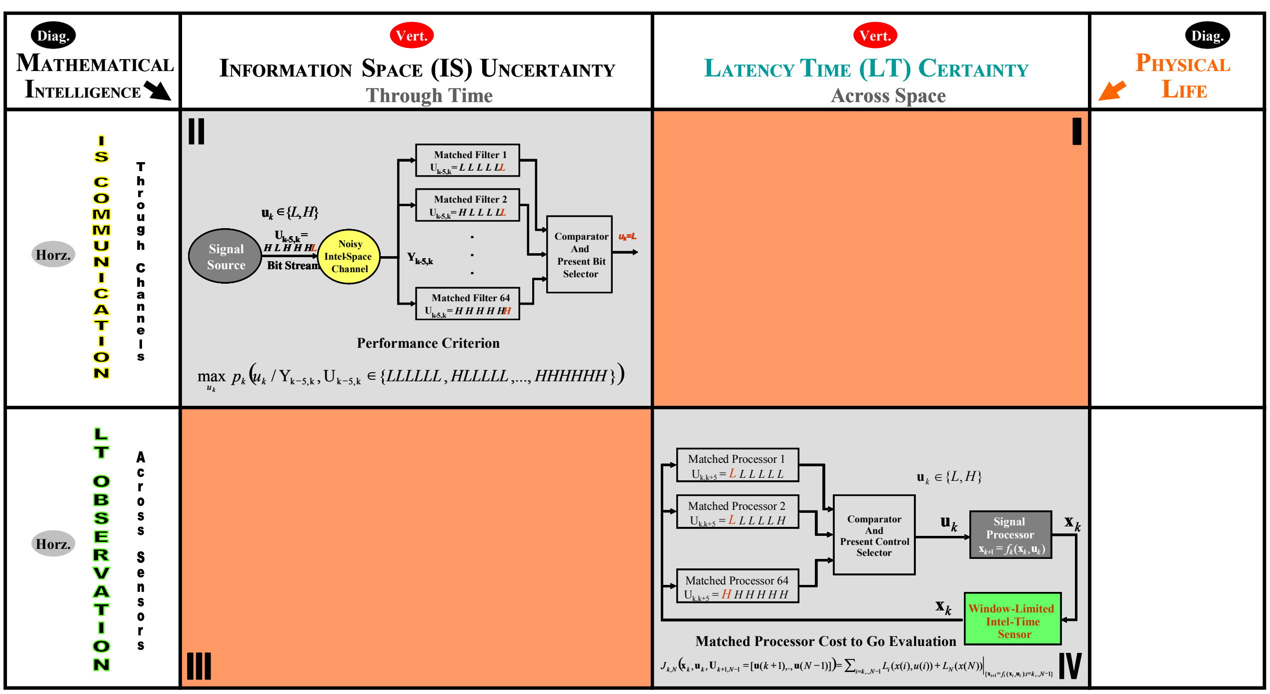


Fig. 6. The LIT revolution display of the IS-uncertainty detection and the LT-certainty quantized control duals.

The detailed matched processors architecture of the quantized control evaluator is given in the LT-certainty/LT-observation quadrant IV of the LIT revolution given in Fig. 6 for a simple case where $\mathbf{u}_k$ is a scalar and $\{\mathbf{q}_1,..,\mathbf{q}_M\}$ consists of the binary set $\{\boldsymbol{L}, \boldsymbol{H}\}$ with $\boldsymbol{L}$ denoting a low voltage and $\boldsymbol{H}$ a high voltage. From this figure it is noted that the matched processors are constructed from the deterministic (or certainty) cost to go expression under the assumption that there are not state or output disturbances associated with the signal processor. Thus this solution approach assumes the separation of the stochastic quantized control problem into two physically independent design problems. One is the control evaluator subsection design problem depicted in the LT-certainty/LT-observation quadrant IV of Fig. 6. The other is the state estimator design problem depicted in the IS-uncertainty/IS-communication quadrant II of Fig. 4 that uses the non-linear state estimator and controlled signal processor structures of Fig. 5 instead of the linear ones of Fig. 3. Moreover, in Fig. 6 it is also shown that the displayed matched processors control evaluation structure of quadrant IV is similar to that of the matched filters solution for the detection of bits in communications displayed in quadrant II. This solution approach to stochastic quantized control problems was first conjectured by the author in 1978 [5]. In essence, this scheme offered a structural-physical LT-certainty/IS-uncertainty duality methodology for the design of quantized control systems that more recently has led him to the discovery of time duals for both information theory and thermodynamics as well as a space dual for the laws of motion in physics [1]-[3], [12].

A close investigation of the matched processors solution given in Fig. 6 reveals a potential dimensionality problem since the number of matched processors needed for the control evaluations is an exponential function of the number of control stages to go. Fortunately, however, this problem is readily addressed by assuming that at each control stage the control sequence to go, i.e., $U_{k+I,N-1}$ (where I is a small integer number, say one, two, etc.) is set to a suitable constant value, e.g., zero. Fig. 7 presents such a scheme where only 2 matched processors, rather than 64, are used since it is assumed that $U_{k+1,k+5}$=[0 0 0 0 0] (notice that in Fig. 6 a total of 64 matched processors must perform evaluations to arrive at an optimum solution). In [5]-[6] is shown that this approach can yield outstanding practical results. Moreover, it is proven for a scalar case in [5], in a more than 50 pages long proof that such a scheme yields an *optimum* solution regardless of the number of control stages to go.

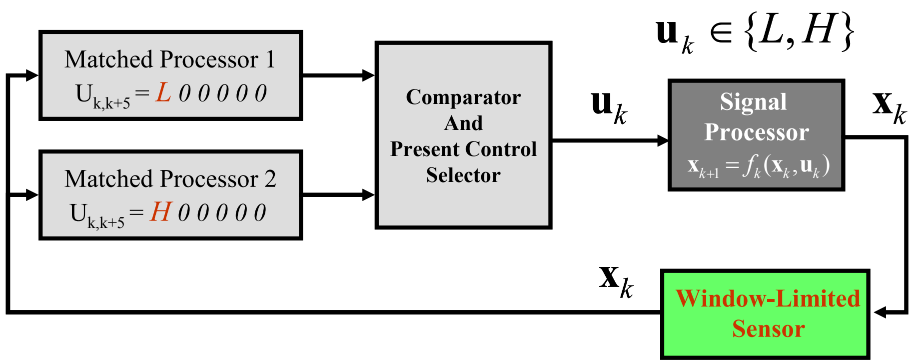


Fig. 7. A Simple Matched Processors Controller That Only Uses Two Matched Processors.

5. Conclusions

In this review paper it has been noted that the conjectured structural-physical LT-certainty/IS-uncertainty duality of the LIT revolution is similar to that advanced by the author in 1978 to address the practical solution of stochastic control problems. This assumed property manifests itself in the LIT revolution by its guidance of system designs via duality structures and performance bounds. The advancement of the four physically independent quadrants of the LIT revolution has further motivated the search for a statistical physics bridge for all of its quadrants. This investigation has given rise to the revelation of a time dual for thermodynamics that has been named lingerdynamics and like thermodynamics has four physical laws that drive the Universe.

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